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CZECHOSLOVAK USE OF THE "URAL 1" COMPUTER FOR GEODETIC PROJECTS

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1. Introduction

In the past years we have been witnessing a gradually increasing application of mathematical machines in various spheres of human activity. Among them the most important are the computers, which enable us, on the basis of a program established beforehand, to solve at a very high speed even such problems as have been practically insolvable with the application of the former aids. The purpose of this article is to inform the reader about the application of one of these machines, the computer "URAL I", in two frequently occuring tasks in geodesy and to describe in outline the programming process. The actual program was made up for the needs of the VUGTK /Vyzkumny Ustav Geodesie Topografie a Kartografie -- Research Institute of Geodesy, Topography and Cartography/ in Prague.

2. Characteristics of the "URAL I" Computer

The "URAL I" computer is a universal electronic machine designed for the solution of various problems of a scientific-technical character. The workers of the scientific-research institute of the Ministry of Precision Machinery of the USSR,

headed by Engineer B. I. Rameyev had in mind two basic criteria in its design; first of all, that the machine be sufficiently accurate and rapid for most current problems, and further that its construction and application be as simple as possible. The result of their work is a computer which processes ten-digit numbers at a speed of 100 operations per minute, and consists of 800 tubes and over 3000 germanium diodes. The individual construction elements, both electronic and non-electronic, are arranged in a block system and are easily replaceable. The use of the machine is facilitated by equipment for manual control and observation of its performance, which is of great importance for the removing (from the program) of errors from the instruction network. Similarly, even the proper compilation of the construction network is further facilitated by the fact that the instruction system of the "URAL I" computer includes in addition to the elementary arithmetical operations a number of special operations (there are 30 basic instructions) which make it possible to make up programs of multiple ramification with a rich logical structure.

The machine consists of the following main parts:

1. The operational unit, performing all arithmetical and logical operations,

2. The synchronizer, which coordinates the performance of the individual blocks of the machine,

3. The memory units:

(a) operational memory on a magnetic drum with a capacity of 1024 complete words;

(b) auxiliary memory on a magnetic tape, serving for preserving the contents of any part of the operational memory (e.g. the group of partial results arrived at during the computation, or a part of the instruction network);

(c) perforated tape memory (standard perforated movie film) which is the carrier of the initial and — together with the printing unit — also of the final information;

4. The control table, which makes it possible for the operator actively to interpose during the computation and manually to insert information into the machine.

5. The outer equipment, consisting of the exit of the information (exit puncher and printer) and of the tape punching and control with the entrance data (keyboard, entrance puncher, and the control unit, serving also for making copies of the perforated tapes).

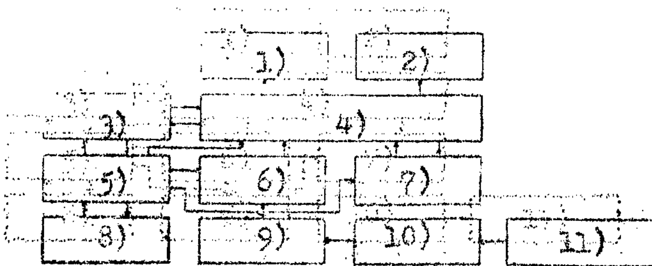

Illustration 1.

Legend.

1) Exit Perforation
2) Printing
3) Operational Memory
4) Operational Unit
5) Synchronizer
6) Perforated Tape
7) Magnetic Tape
8) Control Table
9) Entrance Puncher
10) Control Unit
11) Keybord Unit

The functional relation between the individual parts of the machine is given in Illustration 1.

Note: A more detailed explanation of the fundamental terms pertaining to the theory of the computers can be found, e.g. in the article by S. Malon and O. Valka entitled "The Application of Computers in Geodesy" published in Geodeticky a kartograficky obzor, No 10, 1959.

3. Choice of Method for Calculation of Both Main Geodetic Problems and Its Formulation for the Machine

The choice of a mathematical method for the computer has certain characteristic features by which it differs from the choice of method used in manual or electrical calculating machines. The calculating machines facilitate for the operator the individual numerical operations, but the control of the computation, based on a logical analysis in its course, is reserved to the human brain. The introduction of the computers into the sphere of the computation technique means that even this other, governing component of the computation, goes to the machine. In connection with the elimination of human judgment from the immediate contact with the problem which is being solved and its substitution by a limited number of mechanical operations is the circumstance that many calculating methods suitable for manual calculation are for the computer very uneconomical or even inapplicable. On the other hand, the great speed at which computers work makes it possible to use those

methods which have been practically inaccessible up to now because of the great number of numerical operations involved. From the above it follows that it is necessary to choose such a method as would have a simple logical structure and in which it is not substantial whether the simplicity can be obtained at the price of an increased number of elementary arithmetical operations. This condition is fulfilled especially by several numerical methods of calculation (e.g. numerical integration) which are frequently used in programming. Another favorable fact is that such a method is very easy to program cyclically (multiple repetition of a part or of the entire calculation according to the same program with various entrance values), which in the application of the computers has a great economic meaning. Thereby not only the time necessary for the compilation of the program, but even the length of the instructions is considerably reduced.

The mentioned basic condition of the selection of the mathematical method had to be respected also in the selection of the method for the solution of both of the main geodetic problems.

The formulation of the problem: Assume that on the surface of a rotary ellipsoid of given parameters there are given two points P_1 (φ_1, λ_1,) and P_2 (φ_2, λ_2). These two points are connected by a geodetic curve of a length s whose azimuths in the terminal points will be marked α_1, and α_2. Considering the quantities $\varphi_1, \lambda_1 \alpha_1$, s (or $\varphi_1, \lambda_1, \varphi_2, \lambda_2$,) as known and determining the quantities $\varphi_2, \lambda_2, \alpha_2$, (or α_1, α_2, s) we solve the first (or the second) geodetic problem (Illustration 2).

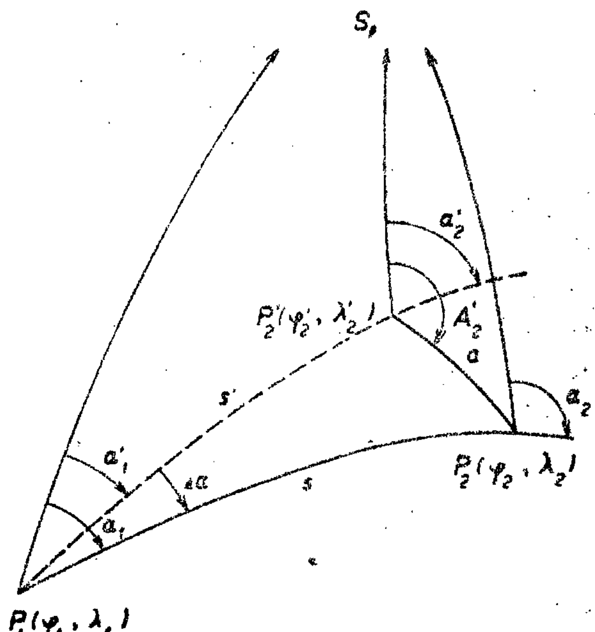

Illustration 2.

3.1 Solution of the First Geodetic Problem

For the solution of the first geodetic problem we selected the numerical integration of the differential equations of the geodetic curve

$$\frac{d\varphi}{ds} = \dot{\varphi} = \frac{\cos\alpha}{M} \qquad \frac{d\lambda}{ds} = \dot{\lambda} = \frac{\sin\alpha}{N\cos\varphi}$$

$$\frac{d\alpha}{ds} = \dot{\alpha} = \frac{\operatorname{tg}\varphi\sin\alpha}{N} \qquad (1)$$

by the method of Runge-Kutt (v. 3)

Note: Another method of solution of both main geodetic problems in the computer is dealt with by H. M. Dufour in the book <u>Resolutions pratiques du probleme des grades geodesiques par l'emploi d'une sphere auxiliare</u>, Paris 1957.

The method of Runge-Kutt makes it possible to determine from the known initial quantities $({}_i\varphi, {}_i\lambda, {}_i\alpha)$ in the beginning of each integrating step the quantities $({}_{i+1}\varphi, {}_{i+1}\lambda, {}_{i+1}\alpha)$ at its end. If h denotes the width of the $(i+1)^{th}$ step, we arrive at the following results applying the Runge-Kutt formula in the solution of the mentioned system of differential equations:

$$\begin{aligned} {}_{i+1}\varphi &= {}_i\varphi + \tfrac{1}{6}({}^\varphi k_1 + 2\,{}^\varphi k_2 + 2\,{}^\varphi k_3 + {}^\varphi k_4) \\ {}_{i+1}\lambda &= {}_i\lambda + \tfrac{1}{6}({}^\lambda k_1 + 2\,{}^\lambda k_2 + 2\,{}^\lambda k_3 + {}^\lambda k_4) \\ {}_{i+1}\alpha &= {}_i\alpha + \tfrac{1}{6}({}^\alpha k_1 + 2\,{}^\alpha k_2 + 2\,{}^\alpha k_3 + {}^\alpha k_4), \end{aligned} \qquad (2)$$

where
$$\begin{aligned} {}^\varphi k_l &= h\,.\,\dot{\varphi}_l \\ {}^\lambda k_l &= h\,.\,\dot{\lambda}_l \quad l = 1, 2, 3, 4 \\ {}^\alpha k_l &= h\,.\,\dot{\alpha}_l \end{aligned}$$

The index means that the respective quantities $(\dot{\varphi}, \dot{\lambda}, \dot{\alpha})$ are to be determined gradually for the arguments of the line of the following table

Table 1.

l	φ	λ	α
1	${}_i\varphi$	${}_i\lambda$	${}_i\alpha$
2	${}_i\varphi + \frac{1}{2}\,{}^\varphi k_1$	${}_i\lambda + \frac{1}{2}\,{}^\lambda k_1$	${}_i\alpha + \frac{1}{2}\,{}^\alpha k_1$
3	${}_i\varphi + \frac{1}{2}\,{}^\varphi k_2$	${}_i\lambda + \frac{1}{2}\,{}^\lambda k_2$	${}_i\alpha + \frac{1}{2}\,{}^\alpha k_2$
4	${}_i\varphi + {}^\varphi k_3$	${}_i\lambda + {}^\lambda k_3$	${}_i\alpha + {}^\alpha k_3$

The numerical integration of the (1) equations consequently consists in our assumption that the length of the s curve is divided into several intervals and from the initial quantities $\varphi_1, \lambda_1, \alpha_1$,

we gradually determine φ, λ, α at the end of each interval until we arrive at the terminal of the curve. The derivation of the mentioned formulas can be found, e.g., in 3.

The formulas of Runge-Kutt solve our problem with an accuracy which decreases with the increasing integrating step h (if we do not take into consideration the errors caused by rounding off). On the basis of solving several trial problems, the value $h_0 = 125$ km has been established, with which in most problems the accuracy specified in paragraph 5 will be obtained. An optimum h_0 applicable to any chosen geodetic curve cannot be established because such a quantity is fundamentally dependent on the position of the curve on the ellipsoid. The fact that the chosen h_0 quantity can be in exceptional cases rather approximate does not affect the calculation, as will become apparent in the following procedure.

The problem is first computed with the integrating step doubled, $2h_0 = 250$ km (on the assumption that the curve is shorter, we hold h being equal to s). The results of this calculation are retained in the operational memory, and in addition to this, they are printed, together with the entrance quantities. After the computation made with this double quantity, the machine divides the step automatically in two and makes another calculation. The results of the second calculation are again printed (for visual appraisal of the accuracy of both calculations) and in addition to this they are compared with the results of the first calculation. If the difference between the two calculations is so small that it guarantees the accuracy of the second calculation (the estimate of accuracy of the method of Runge-Kutt is described, e.g., in 3), the machine either stops or proceeds to the solution of the next problem (see Illustration 4). In case the difference of the two calculations exceeds the established limit, which in current problems usually does not occur, the machine once more splits the step in two and computes it. If there is no agreement of results even after this third calculation, the machine comes to a stop because any further halving could violate the accuracy of the results because of the rounding off. In this last case, further work of the machine is controlled manually according to the character of the obtained results.

Note: In the first two calculations the error caused by rounding off will not show because the inner accuracy of the machine is higher than the accuracy of the entrance and exit data; only in a third calculation the rounding off errors may with long curves exceptionally reach the values mentioned in the paragraph 5.

The above mentioned method has, besides the automatic determination of accuracy, the advantage that it is economical from the viewpoint of the mechanical time. The calculation with the double integrating step is twice as fast as the calculation with the normal step, so that the time necessary for a controlled calculation of longer curves ($s > 250$ km) is shorter than the double

of the time needed for one calculation. This circumstance is especially significant with long curves.

3.2 Solution of the Second Main Geodetic Problem

In selecting a method for the solution of the second main geodetic problem, we looked for one which could use the prepared program for the solution of the first main geodetic problem which was given by the requirement for a program as short and simple as possible. In the following paragraphs we present the basic idea of the method and a list of the formulas used.

From the given coordinates (φ_1, λ_1,) and (φ_2, λ_2) of the points P_1 and P_2 we determine the approximate values s' and α_1' by means of the simplified formulas of the mean width method where we keep in progress only the members of the first order. If we denote (see Illustration 2)

$$\Delta\varphi = \varphi_2 - \varphi_1, \quad \Delta\lambda = \lambda_2 - \lambda_1, \quad \varphi = \frac{1}{2}(\varphi_1 + \varphi_2), \qquad (3a)$$

it follows that

$$\operatorname{tg} \alpha = \frac{N \cos\varphi \,.\, \Delta\lambda}{M \Delta\varphi} \qquad (3b)$$

$$s' = \frac{M \Delta\varphi}{\cos\alpha} = \frac{N \cos\varphi \,.\, \Delta\lambda}{\sin\alpha}, \quad \alpha_1' = \alpha - \frac{\Delta\lambda}{2} \sin\varphi.$$

The mentioned approximation may be made only for such $\Delta\varphi$ and $\Delta\lambda$ in which the above mentioned progresses converge. In the program it is therefore taken into account that for such a distance of the terminal points in which the convergence would become too slow, the machine does not compute according to the formula (3), but rather determines the exit values s' and α_1' by means of solving the spherical triangle P_1, P_2, S_p on a substitute sphere of a radius R = 6370 km. With the approximate exit values, the machine works exactly (i.e. with h = 125 km or h = s' for $s' < 125$ km) according to the program for the solution of the first main geodetic problem and determines in addition to the azimuth also the coordinates of the point P_2', which generally is not identical with the point P_2. The following part of the program computes the corrections $\Delta\alpha$ and Δs by which it is necessary to rectify the exit values α_1' and s' so that the point P_2 be moved to the point P_2. Deriving the formulas for these corrections, we determine first the length a and the azimuth A_2' of the geodetic curve connecting the points P_2' and P_2. To determine them is nothing other than to use the program for the solution of the formula (3) in which instead of coordinates of the point P_1 we use the coordinates of the point P_2' and consequently obtain instead of values s' and α_1' the values a and A_2'. If we mark the reduced length m (or m') of the geodetic

curve s (or s') where (v. /4/)

$$m = s\left(1 - \frac{s^2}{6MN} - \frac{s^4}{120\,(MN)^2}\right) \qquad (4a)$$

and

$$s = s' + \Delta s \qquad \alpha_1 = \alpha_1' + \Delta\alpha, \qquad (4b)$$

we obtain according to Illustration 3 the following approximate plane relations

$$\omega = \alpha_2' + 90^\circ - A_2' \qquad (5)$$

$$\Delta s \doteq a\,.\,\sin\omega + \frac{(a\,.\,\cos\omega)^2}{2m'} \qquad \Delta\alpha \doteq \frac{a\,.\,\cos\omega}{m}$$

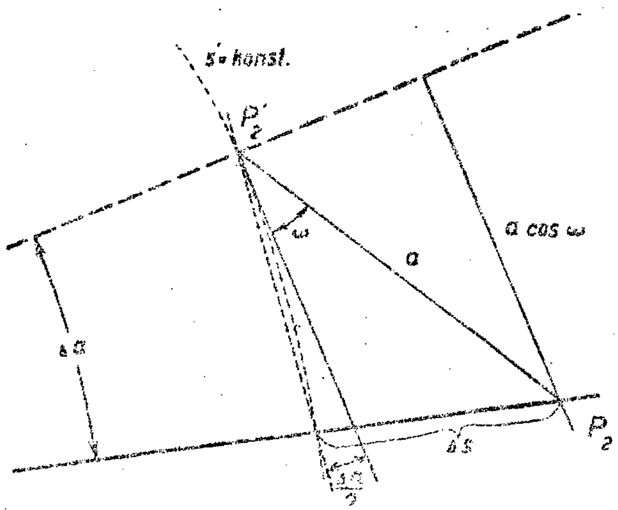

Illustration 3.

The machine now works so that the approximate values α_1' and s' are rectified by the corrections $\Delta\alpha$ and Δs and with the rectified values the first geodetic problem is computed (with a step either identical or reduced). As the experimental calculations showed, it is possible to obtain by this single repetition with not too long curves (1300 km) the accuracy mentioned in Paragraph 5. In case the terminal point of the curve computed with the rectified data fails to coincide, with the required accuracy, with the point

P_2, the program is arranged so that the machine will compute new corrections $\Delta\alpha'$ and $\Delta s'$ and the computation will be repeated. When the machine performs correctly, after the second repetition the terminal points coincide, even with long curves. The machine proceeds then similarly as in the first geodetic problem, i.e., it either stops or passes to the next problem.

Note: It would be possible, of course, by making the formulas (3b), (4a) and (5) more accurate, to achieve the coincidence by a single repetition, but this is not practical from the viewpoint of programming, especially if we take into consideration that problems involving extremely long curves occur only sporadically.

3.3 Schematic Outline of the Program for Solving Both Main Geodetic Problems

The formulas given in the paragraph 3.1 and 3.2 are the basis of a common program for the solution of both of the geodetic problems. The logical structure of the program is presented in the schematic Illustration 4.

Comments on the individual blocks: [in Illustration 4]

3 — the entrance quantities are punched in groups of 10 problems into the individual parts (zones) of the perforated tape. The operational memory, whose capacity is limited, receives only one zone and only after the solution of all the problems contained in it is the next zone introduced.

4,5 — the program is suitable for solution on Krassovsky's, Hayford's or Bessel's ellipsoid. The selection of the ellipsoid constants is done by the machine automatically according to a number code which denotes the selected ellipsoid. This symbol, together with the sign which furnishes the machine with the information whether the first or the second geodetic problem should be computed, are punched in each zone together with the entrance values. In addition to the mentioned ellipsoids, the program may be used for computations on any other rotary ellipsoid whose parameters naturally must be inserted into the operational memory.

9 — if it were necessary to study the course of the curve on the ellipsoid (e.g. for cartographic purposes), it is possible to supplement the program so that the φ, λ, α values are printed at the end, either of each or any chosen integrating step.

10,17 — marks the switches on the control table which can be manually set into two positions. The K_1 switch comes into use in testing stoppage-free performance of the machine and in localizing a possible stoppage. The other switch, K_2, enables us to stop the computation of the respective problem.

14,16 — after pressing the respective bar the computation proceeds according to the schematic picture.

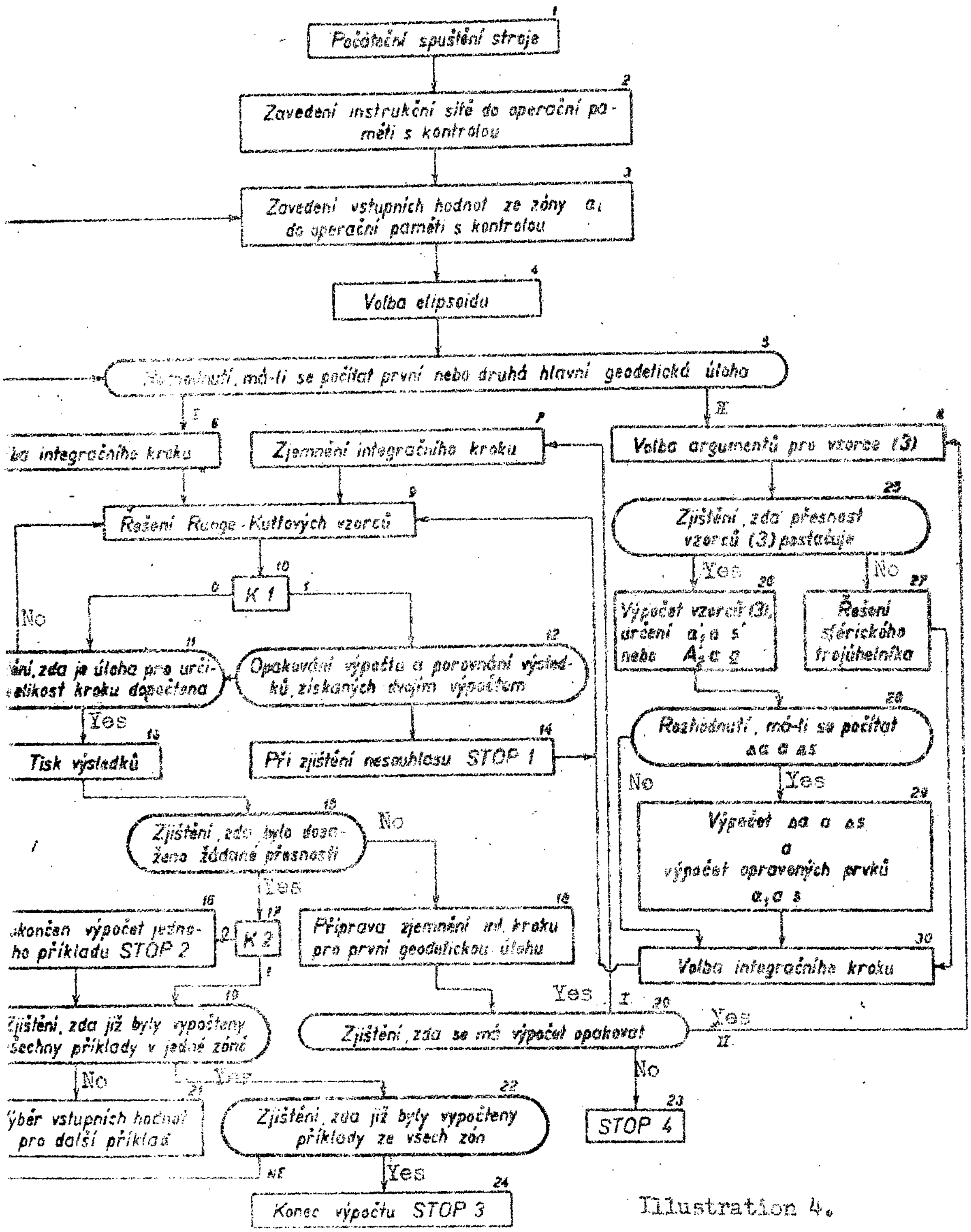

Illustration 4.

Illustration 4. Legend

1. Initial Starting of the Machine
2. Introduction of Instructions into the Operational Memory with Control
3. Introduction of the Entrance Values from the a_1 zone into the Operational Memory with Control
4. Selection of Ellipsoid
5. Decision whether the First or the Second Main Geodetic Problem should be Solved
6. Selection of the Integrating Step
7. Refinement of the Integrating Step
8. Selection of Arguments for the Formulas (3)
9. Solution of the Runge-Kutt Formulas
10. K 1
11. Check whether the Problem for a certain Length of Step has been Completed
12. Repetition of the Calculation and Comparison of the Results obtained by repeated Calculation
13. Printing of Results
14. When Disagreement - STOP 1
15. Check whether required Accuracy has been Obtained
16. Calculation of one Problem completed STOP 2
17. K 2
18. Preparation for Refinement of the Integrating Step for the First Geodetic Problem
19. Check whether all Problems in one Zone have been Completed
20. Check whether the Calculation should be Repeated
21. Selection of Entrance Values for next Problem
22. Check whether Problems from all Zones have been Completed
23. STOP 4
24. End of Computation STOP 3
25. Check whether the Accuray of the (3) Formulas is Sufficient
26. Computation of (3) Formulas, Determination of a_1 a s' or A_2 a $\underline{a}$
27. Spherical Triangle Solution
28. Decision whether Δ a and Δ s should be computed
29. Computation of Δ a Δ s and Computation of Rectified Values a_1 a s
30. Selection of the Integrating Step

The compilation of the rough schematic picture represents the first stage of the programming. The next stage includes its breaking into a detailed schematic picture, which is the basis for the actual distribution of the instruction network. The detailed schematic picture of our problem contains over 200 elementary blocks; the instruction network consists of some 900 one-address instructions.

4. Problems

One of the control problems which were used to test the correctness of the program is the solution of both main geodetic problems for a curve of the following parameters

$$\begin{aligned} \varphi_1 &= 40^\circ 45' 23{,}[illegible]'' & \varphi_2 &= 48^\circ 50' 11{,}0000'' \\ \lambda_1 &= 0^\circ 00' 00{,}0000'' & \lambda_2 &= 76^\circ 18' 40{,}0000'' \\ \alpha_1 &= 53^\circ 47' 20{,}[illegible]335'' & \alpha_2 &= 111^\circ 51' 29{,}3062'' \\ s &= 5\,347\,970{,}613 \text{ m} \end{aligned} \qquad (6)$$

on Hayford's ellipsoid. These parameters were taken from /1/ and will later on be compared with the results received according to the described method on the computer.

4.1 Solution of the First Geodetic Problem

Results of the first computation (h = 250 km)

$$\begin{aligned} \varphi_2 &= 48^\circ 50' 11{,}000_3'' \\ \lambda_2 &= 76^\circ 18' 40{,}006_9'' \\ \alpha_2 &= 111^\circ 51' 29{,}304_6'' \end{aligned} \qquad (7)$$

Results of the second computation ($h = h_0 = 125$ km)

$$\begin{aligned} \varphi_2 &= 48^\circ 50' 11{,}000_3'' \\ \lambda_2 &= 76^\circ 18' 40{,}001_3'' \\ \alpha_2 &= 111^\circ 51' 29{,}307_2'' \end{aligned} \qquad (8)$$

After finishing the second computation, the machine proceeded automatically to the computation of the next problem because the values of (8) have been proved as final (see paragraph 3.1).

The errors of the individual approximations are given in the following table:

Table 2.

	$\delta\varphi_2$	$\delta\lambda_2$	$\delta\alpha_2$
I. ($h = 250$ km)	$-0{,}000_3''$	$-0{,}006_9''$	$0{,}001_6''$
II. ($h = 125$ km)	$-0{,}000_3''$	$-0{,}001_3''$	$-0{,}001_0''$

4.2 Solution of the Second Main Geodetic Problem

Results of the first computation:

4.2 Výpočet druhé hlavní geodetické úlohy

Výsledky prvního výpočtu:

$$\begin{array}{ll} s' = 5\,831\,094{,}46\text{ m} & \varphi_2 = 48°54'25{,}355_3'' \\ \alpha_1' = 53°46'10{,}762_2'' & \lambda_2' = 76°08'22{,}266_7'' \quad (9) \\ & \alpha_2' = 111°41'30{,}842_2'' \end{array}$$

Result of the second computation

$$\begin{array}{ll} s' = 5\,847\,970{,}70\text{ m} & \varphi_2' = 48°50'10{,}995_1'' \\ \alpha_1' = 53°47'20{,}899_1'' & \lambda_2' = 76°18'40{,}004_6'' \quad (10) \\ & \alpha_2' = 111°51'29{,}312_6'' \end{array}$$

Since after the second computation the terminal points had not coincided within the limits of an expected accuracy, the machine once more corrected the s' and α 1' quantities and arrived at the following final results:

$$\begin{array}{ll} s = 5\,847\,970{,}60\text{ m} & \varphi_2 = 48°50'11{,}000_3'' \\ \alpha_1 = 53°47'20{,}893_9'' & \lambda_2 = 76°18'39{,}999_2'' \quad (11) \\ & \alpha_2 = 111°51'29{,}307_2'' \end{array}$$

The errors of the individual approximations are given in Table 3.

Table 3.

	δs	$\delta\alpha_1$	$\delta\alpha_2$
I.	16 876, 15_2 m	$1'10{,}131_4''$	$9'58{,}483_2''$
II.	— 0, 08_7 m	— $0{,}005_6''$	— $0{,}009_3''$
III.	— 0, 01_3 m	— $0{,}000_4''$	— $0{,}001_6''$

4.3 Demonstration of the Printing of the Results

The arc data (φ, λ, α) are inserted into the machine and printed by it in arc units to a scale of $(2\pi)^{-1}$

$$\bar{\alpha} = \frac{\alpha°}{\varrho°} \cdot \frac{1}{2\pi} \qquad (12)$$

The arc measuring system was selected because we may express an angle by means of it much more accurately than by means of angle, the number of digits being same in both cases. The necessity to introduce a scale is caused by the construction requirement with regard to numerical extent of the processed quantities. The scale of the length data expressed in meters is 10^{-7}. It holds true that

$$\bar{s} = \frac{s}{10^7} \qquad (13)$$

The program is prepared so that before the printing of the results of any solution the entrance quantities used in the problem will be printed as well.

The actual figures obtained in the solution of the problem under 4.1 (first main geodetic problem) are given in Illustration 5.

1 1 3 2 1 2 1 9 2	
1 4 9 4 1 4 2 7 0	(14a)
0 0 0 0 0 0 0 0 0	
5 8 4 7 9 7 0 6 2	
1 3 5 6 5 6 6 3 6	
3 1 0 7 1 7 0 5 6	(14b)
2 1 1 9 7 5 3 1 4	
5 8 4 7 9 7 0 6 2	
1 1 3 2 1 2 1 9 2	
1 4 9 4 1 4 2 7 0	(15a)
0 0 0 0 0 0 0 0 0	
5 8 4 7 9 7 0 6 2	
1 3 5 6 5 6 6 3 6	
3 1 0 7 1 7 0 5 8	(15b)
2 1 1 9 7 5 3 1 0	
5 8 4 7 9 7 0 6 2	

Illustration 5.

The data (14a) and (15a) are the entrance quantities, (14b) the results arrived at computing with the step h = 250 km, (15b) the results arrived at computing with the step h = 125 km. The sequence of the data in the individual groups of four is φ, α, λ, s. The decimal point is placed before the first figure from the left. If we want to obtain the results in the conventional form, we have to multiply the printed quantities by the inverted value of the scale.

The angular data are thus rendered as arc values and the length data are given in meters. If we want to express the angular data in degress, the printed quantities will be simply multiplied by 360°. It follows that

$$\bar{\alpha} \,.\, 360^\circ = \frac{\alpha^\circ}{\rho^\circ} \cdot \frac{1}{2\pi} \,.\, 360^\circ = \frac{\alpha^\circ}{\frac{360^\circ}{2\pi}} \cdot \frac{1}{2\pi} \,.\, 360^\circ = \alpha^\circ \qquad (16)$$

and similarly

$$\bar{s} \,.\, 10^7 = \frac{s}{10^7} \,.\, 10^7 = s \qquad (17)$$

5. Conclusion

The program for the solution of both main geodetic problems makes it possible to calculate these on any rotary ellipsoid entirely automatically. Curves up to 10,000 km may be computed with the following accuracy:

The first geodetic problem: the error in the geographical coordinates and the azimuth of the terminal point will not exceed 0.003", practically regardless of the length of the curve.

The second geodetic problem: the error in the length of the curve is less than 10 cm and likewise is not dependent on the length of the curve. The reason is the inaccuracy of the approximation of the points P_2 and P_2' independent of the distance between the points P_1 and P_2. The error in azimuths originates from the same reason and changes with the length of the curve. For each problem its maximum can be established from the relation 0.003" x R ÷ s. For s $>$ R the error is the same as in the first geodetic problem. Certain values of the maximum azimuth error are given in the following table:

Table 4.

s_{km}	100	500	1000	2000	5000	10,000
$\delta\alpha''$	0,191	0,038	0,019	0,010	0,004	0,003

The speed of the computation depends on the length of the curve. For information, let us mention that the computation of the second main geodetic problem for a distance of 1000 km takes approximately 4 minutes. The computation of the first geodetic problem is somewhat shorter — for the same distance it takes about 3 minutes.

The correctness of the program has been proven by solving several control problems on various ellipsoids for curves of various

lengths. The results of these problems agree with the results of mechanical computation with an accuracy which is generally higher than that mentioned above.

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